INVESTIGATING THE ONE-PHOTON ANNIHILATION CHANNEL IN AN e^-e^+ PLASMA CREATED FROM VACUUM IN STRONG LASER FIELDS

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Abstract

It is well known that in the presence of strong external electromagnetic fields many processes forbidden in standard QED become possible. One example is the one-photon annihilation process considered recently by the present authors in the framework of a kinetic approach to the quasiparticle $e^-e^+\gamma$ plasma created from vacuum in the focal spot of two counter-propagating laser beams. In these works the domain of large values of the adiabaticity parameter $\gamma \gg 1$ (corresponding to multiphoton processes) was considered. In the present work we estimate the intensity of the radiation stemming from photon annihilation in the framework of the effective mass model where $\gamma \lesssim 1$, corresponding to large electric fields $E \lesssim E_c = m^2/e$ and high laser field frequencies $\nu \lesssim m$ (the domain characteristic for X-ray lasers of the next generation). Under such limiting conditions the resulting effect is sufficiently large to be accessible to experimental observation.

INTRODUCTION

The planned experiments [1] for the observation of an e^-e^+ plasma created from the vacuum in the focal spot of two counter-propagating optical laser beams with the intensity $I \gtrsim 10^{21}$ W/cm² raises the problem of an accurate theoretical description of the experimental manifestations of the dynamical Schwinger effect [2], see also Refs. [3, 4, 5]. The existing prediction [6] in the domain of strongly subcritical fields $E \ll E_c = m^2/e$ of a considerable number of secondary annihilation photons is not rather convincing because it is based on the S-matrix approach for the description of quasiparticle excitations in the presence of a strong external electric field. In particular, this approach does not take into account vacuum polarization effects. Apparently, an adequate approach for the description of vacuum excitations in strong electromagnetic fields is a kinetic theory in the quasiparticle representation. The simplest kinetic equation (KE) of such type for the e^-e^+ subsystem has been obtained for the case of linearly polarized, time dependent and spatially homogeneous electric fields [2]. Some generalizations of the KE in the fermion sector have been worked out in Refs. [7, 8, 9]. It can be expected, that electromagnetic field fluctuations of the e^-e^+ plasma are accompanied by the generation of real photons

which can be registered far from the focal spot. The first two equations of the BBGKY chain for the photon sector of the e^-e^+ plasma were obtained in [10, 11]. This level is sufficient for the kinetic description of the one-photon annihilation. In the presence of an external field such process is not forbidden [12]. In the works [10, 11] it was shown that the spectrum of the secondary photons in the low frequency domain $k \ll m$ has the character of the flicker noise. In the work [13] the inclusion of vacuum polarization effects in the one-photon radiation spectrum led to an essential change of the photon KE which was investigated in a broad spectral band including the annihilation domain $\nu \sim 2m$. First we have considered the domain of large adiabaticity parameters $\gamma \gg 1$, where the photon radiation from the focal spot turns out to be very small. However, the tendency of the effect to grow for $\gamma \to 1$ has been discovered. This is just the domain of practical interest for parameters of modern lasers.

In the present work the effective mass model is considered which allows to investigate the photon radiation in the domain of rather strong fields not restricted to specific values of the adiabacity parameter. Some crude estimations in the framework of this model [12] lead to an unexpectedly large total photon production intensity.

EFFECTIVE MASS MODEL

We will proceed from the photon kinetic equation

$$\dot{F}(\vec{k}, t) = \frac{e^2}{4k(2\pi)^3} \int^t dt' \int d^3 p\, e^{-i\theta(\vec{p},\vec{p}+\vec{k},\vec{k};t',t)} \times$$
$$\times K(\vec{p}, \vec{p}+\vec{k}, \vec{k}; t', t) f(\vec{p}, t') f(\vec{p}+\vec{k}, t') + c.c. \quad (1)$$

for the one-photon annihilation mechanism taking into account vacuum polarization effects in the low density approximation [13]. In Eq. (1) $F(\vec{k}, t)$ and $f(\vec{p}, t)$ are the photon and electron (positron) distribution functions, respectively, $\vec{k}$ is the wave vector of the radiated photon and

$$\theta(\vec{p}_1, \vec{p}_2, \vec{k}; t', t) = \int_{t'}^{t} d\tau \left[\omega(\vec{p}_1, \tau) + \omega(\vec{p}_2, \tau) - k \right] \quad (2)$$

is the high frequency phase.

The two-time convolution $K(\vec{p}, \vec{p}+\vec{k}, \vec{k}; t', t)$ of the four-spinors is a slowly varying function of its variables and can

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be replaced by its average $K \to K_0 \sim 1$, which is sufficient for coarse estimations.

The effective mass model [12] is based on the approximation

$$\omega(\vec{p}, t) = \sqrt{m^2 + \left(\vec{p} - e\vec{A}(t)\right)^2} \to$$
$$\to \omega_*(p) = \sqrt{m_*^2 + \vec{p}^2}, \qquad (3)$$

with the effective mass defined by the relation

$$m_*^2 = m^2 + e^2 \left\langle \vec{A}^2(t) \right\rangle = m^2 + e^2 E_0^2/2\nu^2 =$$
$$= m^2(1 + 1/2\gamma^2), \qquad (4)$$

where $\langle ... \rangle$ denotes the time averaging operation, ν is the frequency of the periodic laser field and E_0 is its field strength amplitude, $\gamma = (E_c/E_0)(\nu/m)$ is the adiabaticity parameter.

In this approximation the phase (2) becomes monochromatic

$$\theta(\vec{p}_1, \vec{p}_2, \vec{k}; t', t) = \Omega_*(\vec{p}_1, \vec{p}_2, \vec{k})(t - t'), \qquad (5)$$

$$\Omega_*(\vec{p}_1, \vec{p}_2, \vec{k}) = \omega_*(\vec{p}_1) + \omega_*(\vec{p}_2) - k, \qquad (6)$$

i.e. the approximation (3) leads to a suppression of multiphoton processes (it corresponds to the large values of the adiabacity parameter $\gamma \gg 1$) and the mismatch (6) can be compensated by the harmonics of the fermion distribution functions in Eq. (1) only.

The inspection of the fermion distribution function shows, in particular, that it oscillates basically with twice the laser frequency

$$f(\vec{p}, t) = \frac{1}{2} \bar{f}(\vec{p}) \left[1 - \cos(2\nu t)\right]. \qquad (7)$$

The substitution of Eqs. (5) and (7) into the kinetic equation (1) allows to perform the time integration, leading to the appearance of two harmonics in the radiation spectrum only (the 2nd and the 4th),

$$\dot{F}(\vec{k}, t) = -A^{(2)}(\vec{k}) \cos(2\nu t) + A^{(4)}(\vec{k}) \cos(4\nu t), \quad (8)$$

$$A^{(2)}(\vec{k}) = \frac{\pi^2 K_0 \alpha}{2k} \int \frac{d^3 p}{(2\pi)^3} \bar{f}(\vec{p}) \bar{f}(\vec{p} + \vec{k}) \delta\left(2\nu - \Omega_*\right), \quad (9)$$

$$A^{(4)}(\vec{k}) = \frac{\pi^2 K_0 \alpha}{8k} \int \frac{d^3 p}{(2\pi)^3} \bar{f}(\vec{p}) \bar{f}(\vec{p} + \vec{k}) \delta\left(4\nu - \Omega_*\right). \quad (10)$$

It is important that a constant component is absent here, because the mismatch (6) could not be compensated in this case by other sources of the time dependence on the r.h.s. of Eq. (1).[1]

Thus, in the case of the infinite system the solution (8) can be interpreted as "breathing" of the photon subsystem. However, the situation is changed, when the generation of the $e^-e^+\gamma$ plasma is considered in a small spatial domain

of the focal spot with volume $\sim \lambda^3$ due to the vacuum condition of the absence of the $e^-e^+\gamma$ plasma in the initial moment of switching on the laser field. In this case one can expect, that all annihilation photons generated in the first half-period of the field will leave the volume of the system and therefore in the next half-period the reverse process (photon transformation to e^-e^+ plasma) will be impossible. Such a mechanism leads to a pulsation pattern for the photon radiation from the focal spot. It corresponds to introducing the condition of a positive definite photon production rate on the r.h.s of Eq. (8).

For estimations of the amplitudes (9), (10) let us introduce the additional model approximation in the spirit of the model (3)

$$\omega_*(\vec{p} + \vec{k}) \to \omega_*(p, k) = \sqrt{\omega_*^2(p) + k^2} \qquad (11)$$

and the isotropisation condition $\bar{f}(\vec{p} + \vec{k}) \to \bar{f}(p + k)$. The integrals on the r.h.s of Eqs. (9), (10) can then be calculated. For example,

$$A^{(2)}(\vec{k}) = \frac{K_0 \alpha}{4k} \bar{f}(p_0) \bar{f}(p_0 + k) \frac{\omega_*(p_0) \omega_*(p_0, k)}{\omega_*(p_0) + \omega_*(p_0, k)} p_0, \qquad (12)$$

where

$$p_0 = \sqrt{\frac{4\nu^2(k + \nu)^2}{(k + 2\nu)^2} - m_*^2} \qquad (13)$$

is the root of the equation $\Omega_* - 2\nu = 0$. From Eq. (13) it is follows the threshold condition[2]

$$\frac{2\nu(k + \nu)}{k + 2\nu} \geqslant m_* . \qquad (14)$$

This condition is rather nontrivial because the effective mass (4) depends also on ν. The minimal permissible value $\nu = 2m_*$ corresponds to $k = 0$. For the 4th harmonic the threshold value falls to $\nu = m_*$, which is close to the parameters of the XFEL [15].

The $1/k$ - dependence on the r.h.s. of Eq. (12) corresponds to the flicker noise. This feature in the spectrum of radiated annihilation photons has been found first in Ref. [10].

The number of photons with the frequency k lying in the interval $[k, k + dk]$ and radiated from the focal spot with the volume $\lambda^3 = \nu^{-3}$ per time interval is

$$\frac{d^2 N}{dt dk} = \frac{8\pi k^2}{\nu^3} \dot{F}(\vec{k}, t). \qquad (15)$$

The fraction on the r.h.s. of Eq. (12) is a slow function of the frequencies k and ν and for the sake of a preliminary estimation it can be replaced by $m_*/2$. For the 2nd harmonic we then obtain from Eqs. (12) and (15)

$$\frac{d^2 N^{(2)}}{dt dk} = \frac{2\pi \alpha K_0 k m_*}{\nu^3} \bar{f}(p_0) \bar{f}(p_0 + k) p_0 . \qquad (16)$$

[1]This is in contrast to the case $\gamma \gg 1$, where accounting for multiphoton processes in the phase (2) leads to a constant component [13].

[2] A similar effect was found first in the theory describing the absorption of a weak signal by the e^-e^+ plasma created from vacuum [14].

As a representative characteristics of the effectiveness of the radiation from the focal spot domain we will consider the total photon number per time interval,

$$\dot{N}^{(2)} = \frac{2\pi\alpha K_0 m_*}{\nu^3} \int\limits_0^{k_{\max}} dk \; k \; \bar{f}(p_0)\bar{f}(p_0+k)p_0 \; . \quad (17)$$

The electron and positron distribution functions entering here are defined as the solutions of the corresponding non-perturbative kinetic equation [2, 7] describing vacuum creation of e^-e^+ pairs under the action of a strong, time dependent electric field of a standing wave of two counter propagating laser beams. The cutoff parameter $k_{\max} = 2m_*$ is introduced in order to take into account the annihilation photons in the radiation spectrum.

The fermion distribution function $f(\vec{p}, t)$ is a rapidly decreasing function with its maximum in the point $\vec{p} = 0$ [3]. On this basis for a rough estimation one can put $p_0 = 0$ in the arguments of these functions on the r.h.s. of Eq. (17),

$$\dot{N}^{(2)} = \frac{2\pi\alpha K_0 m_*}{\nu^3} \bar{f}(0) \int\limits_0^{k_{\max}} dk \; k \; \bar{f}(k)p_0 \; , \quad (18)$$

where according to Eq. (13)

$$\begin{aligned}
p_0(k) &= \frac{m_*^2}{k+4m_*}\sqrt{48 + 56\frac{k}{m_*} + 15\frac{k^2}{m_*^2}} \simeq \\
&\simeq \frac{m}{4}\sqrt{48 + 56\frac{k}{m_*}}, \quad (19)
\end{aligned}$$

because the small $k_{\max} \ll m_*$ is effective in the integral (18). As the result, we obtain the following order of magnitude estimate

$$\dot{N}^{(2)} \sim \alpha m_* \bar{f}^2(0) \; . \quad (20)$$

For the XFEL parameters $E_0 = 0.24E_c$ and $\lambda = 15$ nm [15] we have according to the kinetic theory in the e^-e^+ sector $\bar{f}(0) \sim 10^{-2}$. From Eq. (20) then follows

$$\dot{N}^{(2)} \sim 10^{17} \text{ s}^{-1} \; . \quad (21)$$

For the 4th harmonic with the oscillation amplitude (10) the threshold for the generation of annihilation photons is lowered (see discussion after Eq. (14)) but the intensity of the photon radiation is also lowered so that the order of magnitude of (21) remains unchanged.

SUMMARY

We have considered the effective mass model [12] which allows a rather simple solution of the kinetic equation describing (in the framework of the one-photon annihilation mechanism) the photon radiation from the focal spot of two counter-propagating laser beams. This simple model leads to considerable depletion of the spectrum of parametric oscillations of the $e^-e^+\gamma$ plasma: only the 2nd and 4th harmonics remain due to the condition of the absence of a constant component in the photon production rate. Thus a compensation of the mismatch (6) is possible by means of these two harmonics only. The domain of applicability of this model is limited to the X-ray domain of the laser radiation. The model suggests a high integral luminosity of $\sim 10^{15}$ photons per sec from the focal spot. Other features of the model are: the $1/k$-behavior in the infrared domain $k \ll m$ (the flicker noise of electrodynamic nature) and the threshold effect. These results are encouraging for a further detailed study of the photon kinetics on the basis of the one-photon annihilation mechanism in the domain of small adiabacity parameters $\gamma \lesssim 1$.

REFERENCES

[1] G. Gregori, D.B. Blaschke, P.P. Rajeev, H. Chen, R.J. Clarke, T. Huffman, C.D. Murphy, A.V. Prozorkevich, C.D. Roberts, G. Röpke, S.M. Schmidt, S.A. Smolyansky, S. Wilks, R. Bingham, High Energy Dens. Phys. 6 (2010) 166.

[2] S.A. Smolyansky, A.V. Prozorkevich, S.M. Schmidt, D. Blaschke, G. Röpke and V.D. Toneev, Int. J. Mod. Phys. E 7 (1998) 515.

[3] D.B. Blaschke, A.V. Prozorkevich, G. Röpke, C.D. Roberts, S.M. Schmidt, D.S. Shkirmanov and S.A. Smolyansky, Eur. Phys. J. D 55 (2009) 341.

[4] F. Hebenstreit, R. Alkofer and H. Gies, Phys. Rev. D 78 (2008) 061701.

[5] R. Yaresko, M.G. Mustafa and B. Kämpfer, Phys. Plasmas 17 (2010) 103302.

[6] D.B. Blaschke, A.V. Prozorkevich, C.D. Roberts, S.M. Schmidt and S.A. Smolyansky, Phys. Rev. Lett. 96 (2006) 140402.

[7] A.V. Filatov, A.V. Prozorkevich and S.A. Smolyansky, Proc. of SPIE v6165, (2006) 616509.

[8] V.N. Pervushin, V.V. Skokov, Int. J. Mod. Phys. A, 20, (2005) 5689.

[9] S.A. Smolyansky, A.V. Reichel, D.V. Vinnik, S.M. Schmidt, in: "Progress in Nonequilibrium Green's Functions 2", World Scientific, Singapore , p.384 (2003).

[10] D.B. Blaschke, S.M. Schmidt, S.A. Smolyansky and A.V. Tarakanov, Phys. Part. Nucl. 41 (2010) 1004.

[11] D.B. Blaschke, G. Röpke, S.M. Schmidt, S.A. Smolyansky and A.V. Tarakanov, arXiv:1006.1098 [physics.plasm-ph].

[12] V.I. Ritus, Trudy FIAN SSSR, 111 (1979) 5.

[13] S.A. Smolyansky D.B. Blaschke, A.V. Chertilin G. Röpke, A.V. Tarakanov, arXiv:1012.0559 [physics.plasm-ph].

[14] D.B. Blaschke, S.V. Ilyin, A.D. Panferov, G. Röpke and S.A. Smolyansky, Contrib. Plasma Phys. 49 (2009) 602.

[15] A. Ringwald, Phys. Lett. 510, (2001) 107.